Correspondence

Tables of Maximally Flat Impedance-Transforming Networks of Low-Pass-Filter Form

In previous papers, Szentirmai [1] and Matthaei [2] present design theory for synthesis of lumped-element Chebyshev impedance transforming networks. In the paper of Matthaei [2] extensive tables of element values for the impedance-transforming networks are also presented. These networks are of low-pass ladder form consisting of series inductances and shunt capacitances. They give impedance match in the Chebyshev sense between resistor terminations of arbitrary ratio (designs with resistor termination ratios from 1.5 to 50 are tabulated). The responses of these networks have moderately high attenuation at dc (the amount of attenuation depends on the termination ratio); their attenuation falls to a very low level in the impedance-matching band, and then rises monotonically and steeply above the operating band in a manner typical of low-pass filters. The impedance-transforming networks can be realized in lumped-element form for low-frequency applications, and in semi-lumped-element form (using short sections of transmission line of alternating high and low impedances) at microwave frequencies.

In this correspondence the design tables of Matthaei are extended to include impedance-matching filter designs having a maximally-flat transmission characteristic in the matching band. Figure 1 shows the general form of the impedance-transforming structures under consideration. It should be noted that the structure is of the form of a conventional low-pass filter structure. The main difference between these structures and those of conventional low-pass filters is that conventional low-pass filters have terminating resistors of equal (or nearly equal) sizes at each end. For the filters discussed here, the terminating resistors may be of radically different size, which means there will be a sizable reflection loss at zero frequency. As a result of this sizable attenuation $L_{A_{dc}}$ at zero frequency, the transmission characteristics of maximally flat filters of this type have the form in Fig. 2. The maximally flat matching band extends from the lower and upper frequencies of 3-dB attenuation, ω_a' and ω_b', respectively.[1]

Note that the matching band is not symmetrical about ω_0', the frequency of maximum flatness. However, for narrow to moderate bandwidths, the matching band reasonably approximates a symmetrical response in the sense that $\omega_m' \approx \omega_0'$. Above ω_b', the attenuation rises steeply in a manner typical of low-pass filter structures. The attenuation L_A indicated in Fig. 2 is transducer attenuation expressed in decibels, i.e., it is the ratio of the available power of the generator to the power delivered to the load, expressed in decibels.

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[1] The attenuation at ω_a' and ω_b' is in fact 3.0103 dB. For the cases where $L_{A_{dc}}$ is less than 3.01 dB, ω_a' is undefined.

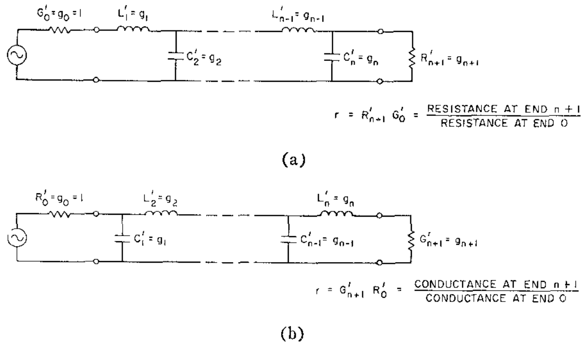

Fig. 1. Definition of normalized prototype element values for impedance-transforming networks of low pass filter form. (The tabulated element values are normalized so that $g_0 = 1$ and $\omega_b' = 1$, as in Fig. 2.)

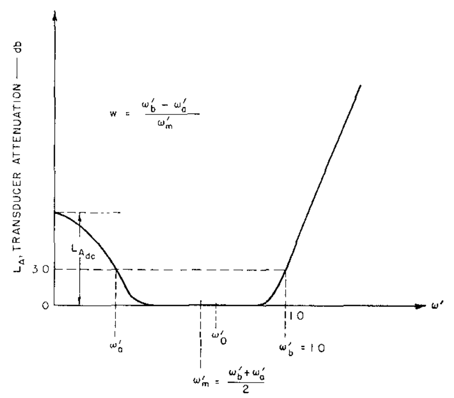

Fig. 2. Definition of response parameters for low-pass impedance-transforming filters. (The frequency scale for the tabulated design is normalized so that $\omega_b' = 1$, as indicated above.)

Parameters of the Attenuation Characteristics

The frequency scale of the networks tabulated herein has been normalized as indicated in Fig. 2 so that

$$\omega_b' = 1 \tag{1}$$

where ω_b' is the upper frequency of 3-dB attenuation, and

$$\omega_m' = \frac{\omega_b' + \omega_a'}{2} \tag{2}$$

where ω_m' is the arithmetic mean of the upper and lower frequencies of 3-dB attenuation.[2] The frequency variables and element values used in the normalized prototype circuits will be primed to indicate that they are normalized, and corresponding unprimed quantities will be reserved for the same parameters scaled to suit specific applications. With the normalization in (1) and (2), the fractional bandwidth w is given by[3]

$$w = \frac{\omega_b' - \omega_a'}{\omega_m'} \tag{3}$$

and the lower 3-dB frequency is given by

$$\omega_a' = 1 - w\omega_m' \tag{4}$$

or

$$\omega_a' = \frac{1 - \frac{w}{2}}{1 + \frac{w}{2}}. \tag{5}$$

The attenuation $L_{A_{dc}}$ at zero frequency is given by

$$L_{A_{dc}} = 10 \log_{10} \frac{(r+1)^2}{4r} \text{dB} \tag{6}$$

where r is again the impedance or admittance transformation ratio.

In some cases it will be desired to determine the attenuation accurately over a range of frequencies, possibly for making use of the strong attenuation band of this type of structure above frequency ω_b'. The attenuation characteristic in Fig. 2 is given by the expression

$$L_A(\omega') = 10 \log_{10} \{1 + A[(\omega'^2 - \omega_0'^2)]^n\} \tag{7}$$

where n is even and equal to the number of reactive elements in the impedance-transforming filter. The frequency ω_0' at which the attenuation is maximally flat is given by

$$\omega_0' = \left\{1 + \sqrt[n]{\frac{4r}{(r-1)^2}}\right\}^{-1/2}. \tag{8}$$

The constant A is related to ω_0' and the impedance ratio r by

$$A = \frac{(r-1)^2}{4r} (\omega_0')^{-2n}. \tag{9}$$

The fractional bandwidth, w, is given in graphical form in Fig. 3. Note in Fig. 3 that w is undefined for r less than 5.83, since in these cases $L_{A_{dc}}$ is less than 3 dB. (Use of the graph of Fig. 3 will be explained by means of an example given later.)

[2] For $L_{A_{dc}}$ less than 3 dB, ω_m' is undefined.

[3] For $L_{A_{dc}}$ less than 3 dB, w is undefined.

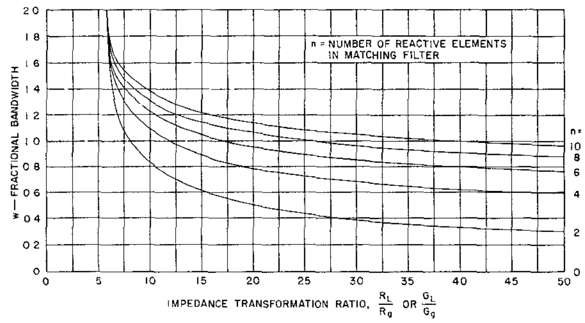

Fig. 3. Fractional bandwidth vs. impedance transformation ratios.

Table 1

ω_0' VS. r AND n

r / n	2	4	6	8	10
1.5	0.41173	0.55785	0.60876	0.63401	0.64901
2.0	0.51108	0.61064	0.64359	0.65983	0.66948
2.5	0.56721	0.63862	0.66194	0.67344	0.68028
3.0	0.60500	0.65709	0.67406	0.68244	0.68743
4.0	0.65465	0.68125	0.68996	0.69428	0.69686
5.0	0.68712	0.69718	0.70050	0.70216	0.70315
6.0	0.71071	0.70891	0.70831	0.70801	0.70783
8.0	0.74368	0.72568	0.71955	0.71646	0.71460
10.0	0.76635	0.73755	0.72757	0.72252	0.71947
15.0	0.80237	0.75727	0.74106	0.73275	0.72771
20.0	0.82458	0.77012	0.74997	0.73956	0.73321
25.0	0.84017	0.77956	0.75660	0.74465	0.73733
30.0	0.85195	0.78699	0.76186	0.74870	0.74062
40.0	0.86896	0.79821	0.76990	0.75493	0.74570
50.0	0.88092	0.80655	0.77595	0.75964	0.74954

Table 2

ELEMENT VALUES g_k VS. r FOR n = 2

r	g_1	g_2
1.5	1.71741	1.14494
2.0	1.95664	0.97832
2.5	2.15923	0.86369
3.0	2.33754	0.77918
4.0	2.64575	0.66144
5.0	2.91069	0.58214
6.0	3.14626	0.52438
8.0	3.55765	0.44471
10.0	3.91466	0.39147
15.0	4.66326	0.31088
20.0	5.28623	0.26431
25.0	5.83095	0.23324
30.0	6.32095	0.21070
40.0	7.18673	0.17967
50.0	7.91620	0.15892

Table 3

ELEMENT VALUES g_k VS. r FOR n = 4

r	g_1	g_2	g_3	g_4
1.5	1.17575	1.57792	2.36688	0.78383
2.0	1.33862	1.41661	2.83323	0.66931
2.5	1.45526	1.30511	3.26278	0.58210
3.0	1.54785	1.22174	3.66522	0.51595
4.0	1.69225	1.10269	4.41076	0.42306
5.0	1.80457	1.01973	5.09863	0.36091
6.0	1.89737	0.95736	5.74417	0.31623
8.0	2.01683	0.86783	6.94268	0.25585
10.0	2.16612	0.80511	8.05105	0.21661
15.0	2.39248	0.70416	10.56247	0.15950
20.0	2.56209	0.64144	12.82873	0.12810
25.0	2.69962	0.59721	14.93037	0.10798
30.0	2.81624	0.56368	16.91018	0.09387
40.0	3.00878	0.51508	20.60321	0.07522
50.0	3.16591	0.48068	24.03380	0.06332

Table 4

ELEMENT VALUES g_k VS. r FOR n = 6

r	g_1	g_2	g_3	g_4	g_5	g_6
1.5	0.90649	1.50478	2.29776	1.53184	2.25717	0.60432
2.0	1.01262	1.43227	2.58638	1.29319	2.86453	0.50631
2.5	1.08328	1.37541	2.83408	1.13363	3.43853	0.43331
3.0	1.13685	1.32997	3.05407	1.01802	3.98991	0.37895
4.0	1.21669	1.26089	3.43742	0.85936	5.04356	0.30417
5.0	1.27612	1.20972	3.76900	0.75380	6.04859	0.25522
6.0	1.32369	1.16949	4.06475	0.67746	7.01693	0.22062
8.0	1.39778	1.10883	4.58216	0.57277	8.87062	0.17472
10.0	1.45493	1.06406	5.03129	0.50313	10.64057	0.14549
15.0	1.55926	0.98749	5.97114	0.39808	14.81236	0.10395
20.0	1.63438	0.93666	6.75001	0.33730	18.73328	0.08172
25.0	1.69362	0.89911	7.42817	0.29713	22.47764	0.06774
30.0	1.74279	0.86957	8.03585	0.26786	26.08704	0.05809
40.0	1.82203	0.82496	9.10416	0.22760	32.99849	0.04555
50.0	1.88508	0.79197	10.03590	0.20072	39.59849	0.03770

Table 5

ELEMENT VALUES g_k VS. r FOR n = 8

r	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
1.5	0.74087	1.40995	2.05938	1.66099	2.49148	1.37291	2.11490	0.49392
2.0	0.81439	1.38835	2.23239	1.47790	2.95580	1.11619	2.77668	0.40720
2.5	0.86126	1.36553	2.37027	1.35099	3.37746	0.94811	3.41381	0.34451
3.0	0.89584	1.34524	2.48706	1.25598	3.76794	0.82902	4.03569	0.29862
4.0	0.94606	1.31179	2.68086	1.12028	4.48113	0.67021	5.24716	0.23652
5.0	0.98253	1.28532	2.84053	1.02576	5.12882	0.56811	6.42657	0.19651
6.0	1.01122	1.26357	2.97786	0.95479	5.72871	0.49631	7.58141	0.16854
8.0	1.05510	1.22929	3.20852	0.85308	6.82462	0.40106	9.83427	0.13189
10.0	1.08835	1.20283	3.40046	0.78201	7.82008	0.34005	12.02828	0.10884
15.0	1.14784	1.15526	3.78222	0.66816	10.02244	0.25215	17.32888	0.07652
20.0	1.18981	1.12199	4.08218	0.59787	11.95741	0.20411	22.43971	0.05949
25.0	1.22245	1.09648	4.33352	0.54860	13.71510	0.17334	27.41201	0.04890
30.0	1.24926	1.07585	4.55208	0.51144	15.34328	0.15174	32.27546	0.04164
40.0	1.29196	1.04369	4.92325	0.45795	18.31782	0.12308	41.74756	0.03230
50.0	1.32550	1.01909	5.23531	0.42039	21.01954	0.10471	50.95444	0.02651

Table 6

ELEMENT VALUES g_k VS. r FOR n = 10

r	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1.5	0.62715	1.31379	1.86515	1.69944	2.35274	1.56847	2.51899	1.24182	1.96987	0.41861
2.0	0.68114	1.31937	1.97242	1.57841	2.65761	1.32880	3.15668	0.98609	2.63816	0.34071
2.5	0.71439	1.31581	2.05366	1.49074	2.92117	1.16846	3.72671	0.82139	3.28901	0.28584
3.0	0.73849	1.31032	2.11973	1.42297	3.15623	1.05207	4.26876	0.70653	3.93018	0.24622
4.0	0.77290	1.29881	2.22519	1.32259	3.56714	0.89186	5.29022	0.55627	5.19479	0.19326
5.0	0.79749	1.28830	2.30897	1.24983	3.92437	0.78487	6.24901	0.46177	6.44102	0.15952
6.0	0.81663	1.27899	2.37918	1.19344	4.24348	0.70725	7.16047	0.39652	7.67344	0.13612
8.0	0.84558	1.26331	2.49390	1.10962	4.80309	0.60039	8.87680	0.31173	10.10589	0.10571
10.0	0.86728	1.25048	2.58674	1.04865	5.28993	0.52899	10.48632	0.25867	12.50418	0.08674
15.0	0.90563	1.22603	2.76557	0.94609	6.31108	0.42074	14.19116	0.18437	18.38974	0.06038
20.0	0.93237	1.20796	2.90155	0.87922	7.15914	0.35796	17.58407	0.14508	24.15838	0.04662
25.0	0.95299	1.19360	3.01289	0.83044	7.89853	0.31594	20.76068	0.12051	29.83899	0.03812
30.0	0.96982	1.18166	3.10799	0.79247	8.56169	0.28539	23.77389	0.10360	35.44879	0.03233
40.0	0.99644	1.16250	3.26626	0.73583	9.72865	0.24322	29.43279	0.08166	46.49869	0.02491
50.0	1.01720	1.14738	3.39648	0.69449	10.74733	0.21495	34.72410	0.06793	57.36778	0.02035

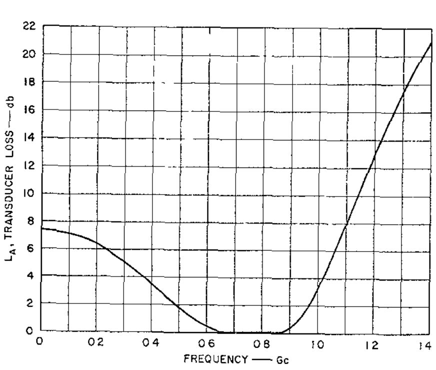

Fig. 4. Transmission response for example in text where $r=20$, $n=4$.

For convenience, ω_0' in (8) is given in tabular form in Table 1 for several values of r and n.

Tables of Prototype Element Values[4]

Tables 2 to 6 (p. 694) give element values for prototype maximally flat impedance-transforming networks for $n=2$, 4, 6, 8, and 10 reactive elements. After the designer has arrived at values for r, $\mathcal{W}$, and n, the normalized element values can be obtained from the tables. Since the networks presented in Tables 2 through 6 are antimetric [3], i.e., half of the network is the inverse of the other half, only half of the network element values need be presented; the remaining elements can be computed from single equations [2]. However, for the convenience of the reader, all element values of the networks are presented in Tables 2 through 6.[5]

Example

A numerical example will serve to demonstrate the use of Tables 2 through 6 and Fig. 3, and Table 1. Suppose that a designer desires a maximally flat impedance-transforming network for an $r=20$ impedance ratio, over the band from 500 to 1000 Mc/s. The required fractional bandwidth is given by

$$w = \frac{f_b - f_a}{f_m} = \frac{2(f_b - f_a)}{f_b + f_a} \tag{10}$$

which for this example gives

$$w = \frac{2(1000 - 500)}{1000 + 500} = 0.667.$$

From Fig. 3, it is found that this value of fractional bandwidth and impedance ratio lies between the $n=2$ and $n=4$ curves, so that $n=4$ reactive elements are necessary. (Two reactive elements would give a fractional bandwidth of only 0.5.) This will give an operating bandwidth somewhat larger than is actually required ($w=0.79$), which is often desirable.

Next, from Table 3, for $n=4$ reactive elements, the element values

$$g_1 = 2.56209$$
$$g_2 = 0.64144$$
$$g_3 = 12.82873$$
$$g_4 = 0.12810$$

are obtained; and from Table 1 [or (9)] ω_0' is found to be 0.77012. The computed transmission response of the network is graphed in Fig. 4.

Scaling of the Normalized Design

After a designer has selected a normalized design, the element values required for a specific application are easily determined by scaling. Let R be the desired resistance level of one of the terminations, while R' is the corresponding resistance of the normalized design. Similarly, let ω_b be the radian frequency of the upper 3-dB frequency of the desired operating band, while $\omega_b'=1$ is the corresponding frequency for the normalized design. Then the scaled element values are computed using

$$R_k = R_k' \left(\frac{R}{R'}\right) \tag{11}$$

$$C_k = C_k' \left(\frac{\omega_b'}{\omega_b}\right) \frac{R'}{R} \tag{12}$$

$$L_k = L_k' \left(\frac{\omega_b'}{\omega_b}\right) \frac{R}{R'} \tag{13}$$

where R_k', C_k', and L_k' are for the normalized design and R_k, C_k, and L_k are for the scaled design.

Acknowledgment

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[4] The derivation of the tables is given in Cristal, et al. [4].

[5] The element values were obtained by a continued fraction expansion of the input impedance of the network. Because of a loss of significant digits in the continued fraction expansion, the element values for second half of the network as given in the tables may be in error in the fourth decimal place. In those cases where the error is significant the element values of the second half of the network should be obtained from the element values of the first half of the network by the relationships given in Matthaei [2].

Tables of Stub Admittances for Maximally Flat Filters Using Shorted Quarter-Wave Stubs

Consider a symmetrical filter consisting of lossless shorted quarter-wave stubs spaced a quarter wavelength apart on a uniform, lossless line. The tables given here list the normalized characteristic stub admittances k_r necessary for a maximally flat response.

The insertion loss, when the filter is inserted between a generator and a load, both of which have real admittances equal to the characteristic admittance of the transmission line, is given by the relation

$$\frac{P_0}{P_L} = 1 + K_n \frac{\cos^{2n}\theta}{\sin^2\theta} \tag{1}$$

where n is the number of shorted stubs of length l,

$$\theta = 2\pi l/\lambda \tag{2}$$

$$K_n = \left(\frac{k_1(k_2+2)\cdots(k_n+2)}{2}\right)^2 \tag{3}$$

and where $k_r = Y_{0r}/Y_0$ is the normalized characteristic admittance of the rth stub.

For example, in a two-stub filter, $n=2$, and symmetry demands that the characteristic impedances of the two stubs be equal, hence

$$\frac{1}{k_1} = \frac{1}{k_2}.$$

The insertion loss is given by (1),

$$\frac{P_0}{P_L} = 1 + \frac{[k_1(k_1+2)]^2\cos^4\theta}{4\sin^2\theta}$$
$$K_2 = [k_1(k_1+2)]^2/4.$$

The following tables give 10 log K_n, and the required normalized characteristic admittances of the stubs for various practical values up to ten stubs. Since the filters are symmetrical, only the values for the first half of the filter are tabulated.

Three-Stub Filter

10 log K_3	k_1	k_2
−12.728	0.100	0 200
− 0.944	0.300	0.600
+ 5.46	0.500	1.000
+10.138	0.700	1 400
+15.56	1.000	2 000
+21.156	1.400	2.800
+27.604	2.000	4.000
+31.904	2.5	5.0
+35.563	3.0	6 0

Four-Stub Filter

10 log K_4	k_1	k_2
− 5.17	0.1	0.292
+ 3.253	0.2	0.571
+13.329	0.4	1.109
+25.668	0.8	2.141
+35.909	1.3	3.395
+44.873	1 9	4.877
+56.734	3 0	7.568

Five-Stub Filter

10 log K_5	k_1	k_2	k_3
+ 3.452	0.100	0.366	0.532
13.577	0.200	0.694	0.989
20.523	0.300	1.005	1.410
26.002	0.400	1.304	1.808
30.601	0.500	1.596	2.193
38.16	0.700	2.166	2.933
44.324	0.900	2.724	3.648
54.172	1.300	3.819	5.038
66.970	2.000	5 702	7.403
77.874	2.800	7.829	10.058

Six-Stub Filter

10 log K_6	k_1	k_2	k_3
+13.378	0.100	0.419	0 755
25.469	0.200	0.774	1.329
33.805	0.300	1.105	1 838
40.388	0.400	1.422	2.314
50.721	0.600	2.031	3.207
58.863	0.800	2.622	4.055
65.668	1.0	3.202	4.878
76.755	1.4	4.343	6.487
85.687	1 8	5.468	8.045
96 571	2.4	5 141	10.359

Seven-Stub Filter

10 $\log_{10} K_7$	k_1	k_2	k_3	k_4
24.63	0 1	0.4556	0 9269	1 1425
38.78	0 2	0.8259	1 5687	1 8856
56.24	0 4	1.4949	2.6514	3 1129
77.83	0.8	2.7308	4.5506	5.2396
85.77	1.0	3.3269	5.4458	6.2379
104.521	1.6	5.0774	9.0398	9.1249
125.521	2 6	7.9395	12.2306	13.7822

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